Exponents and Radicals

Product Rule for Exponents

$a^n \cdot a^m = a^{m+n}$

Simplify:

$2^1 \cdot 2^0 \quad = 2^{(1+0)} = 2$

$5^2 \cdot 5^0 \quad = 5^{(2+0)} = 5^2$

Zero Exponent

If a is any non zero real number then $a^0 = 1$.

(Note: 0^0 is not a number)

Simplify:

$2^1 \cdot 2^{-1} \quad = 2^0 = 1$

$5^2 \cdot 5^{-2} \quad = 5^0 = 1$

Negative exponents

For any nonzero number a,

$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

Quotient Rule for exponents

$\frac{a^n}{b^m} = a^{n-m}$

Simplify the following and write the result using only positive exponents

$\frac{x^5}{x^2} \quad = \quad x^3$

$\frac{2\,x^{-3}\,y}{4\,x^{-5}y^3} \quad = \quad \frac{x^2}{2\,y^2}$

Power Rules for Exponents

$(a^m)^n = a^{mn} \qquad (ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Simplify:

$(a^2)^{-4} \quad = a^{-8} = \frac{1}{a^8}$

$(a\,b)^4\,a^{-2}\,b^3 \quad = \quad a^4 b^4 a^{-2} b^3 = a^2 b^7$

Simplify:

$$\frac{(-5\,y^3\,z^4)^2}{10\,(y^{-4}\,z^2)^{-3}} = \frac{25\,y^6\,z^8}{10\,y^{12}\,z^{-6}} = \frac{5\,z^{14}}{2\,y^6}$$

Radical Notation:

If all of the indicated roots are real numbers, then:

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m \text{ or } a^{m/n} = \sqrt[n]{a^m}$$

Simplify:

$$9^{3/2} = \left(\sqrt{9}\right)^3 = 9\sqrt{9} = 27$$

$$(-8)^{2/3} = \left(\sqrt{-8}\right)^2 = (-2)^2 = 4$$

Simplify:

$$\frac{(x^2\,y^5)^{-1/4}}{(x^{-3}\,y^2)^{1/6}} = \frac{x^{-\frac{1}{2}}\,y^{-\frac{5}{4}}}{x^{-\frac{3}{6}}\,y^{\frac{2}{6}}} = \frac{\cancel{x^{-\frac{1}{2}}}\,y^{-\frac{5}{4}}}{\cancel{x^{-\frac{1}{2}}}\,y^{\frac{1}{3}}}$$

$$= \frac{1}{y^{\left(\frac{1}{3}+\frac{5}{4}\right)}} = \frac{1}{y^{\frac{19}{12}}}$$

Simplifies Radical Expressions:

1. The radicand has no factors raised to a power greater than or equal to the index.
2. The radicand has no fractions.
3. No denominator contains a radical.
4. Exponents in the radicand and the index have no common factors other than 1.

Properties of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then:

1) $\sqrt[n]{a^n} = |a|$ when n is even.

2) $\sqrt[n]{a^n} = a$ when n is odd.

3) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

4) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

5) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Simplify: $\sqrt{44} = \sqrt{4 \cdot 11} = 2\sqrt{11}$

Simplify: $\sqrt{16\,x^4\,y^2}$ $= 4x^2|y|$

Simplify: $\sqrt{\frac{2\,x^3}{27}}$ $= \dfrac{\sqrt{2}\,\sqrt{x^3}}{\sqrt{27}} = \dfrac{\sqrt{2}\,x\sqrt{x}}{3\sqrt{3}}$ $(x \geq 0)$

Simplify: $\sqrt[3]{\frac{16\,x^4}{9}}$ $= \dfrac{2x\sqrt[3]{2x}}{\sqrt[3]{9}}$

$= \dfrac{2x\sqrt[3]{2x}}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{2x\sqrt[3]{6x}}{3}$

Simplify: $\frac{8}{\sqrt{3}+\sqrt{7}}$ $= \dfrac{8}{(\sqrt{3}+\sqrt{7})} \cdot \dfrac{(\sqrt{3}-\sqrt{7})}{(\sqrt{3}-\sqrt{7})}$

$= \dfrac{8(\sqrt{3}-\sqrt{7})}{3-7} = -2(\sqrt{3}-\sqrt{7})$